# LEFT-HANDED NEUTRINO DISAPPEARANCE PROBE OF NEUTRINO MASS AND CHARACTER

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**Abstract**

We explore the sensitivity to a non vanishing neutrino mass offered by dynamical observables, i.e., branching ratios and polarizations. The longitudinal polarization in the C.M. frame decreases by a 4% for $D^+ \to \tau^+ \nu_\tau$ and $m_{\nu_\tau} = 24$ MeV. Taking advantage of the fact that the polarization is a Lorentz variant quantity, we study the polarization effects in a boosted frame. By means of a neutrino beam, produced by a high velocity boosted parent able to flip the neutrino helicity, we find that an enhanced left-handed neutrino deficit, induced by a Wigner rotation, appears.

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The most direct searches for neutrino mass are the experiments which seek a kinematical consequence of this mass in some physical processes [1]. Nevertheless, for massive neutrinos there are noticeable dynamical effects that affect the predictions for physical observables, like transition probabilities and polarizations [2]. In this work we explore the sensitivity to a non-vanishing neutrino mass offered by observables which are modified by these two facts:

a) when $m_\nu \neq 0$ definite chirality in the weak interaction does not mean definite helicity;

b) when both $m_\nu \neq 0$ and the presence of a boost, helicity is not Lorent invariant and the transformed state of an helicity state is a linear combination of different helicity states.

The second fact implies, in particular, that Lorentz variant observables, like the components of polarization, through the so-called "rotation of the spin in a Lorentz transformation" [3], should be modified from the massless case result by enhancement factors associated to the boost momentum. These ideas are applied here to the pure-leptonic decays of pseudoscalar mesons $P \longrightarrow l \ \nu_l$. The dynamical mass-effects are analysed in the rates and the components of polarization in the final state.

Upper bounds on the $m_{\nu_\mu}$, the mass of $\nu_\mu$, come from studies of the decay $\pi \longrightarrow \mu \ \nu_\mu$ using pions at rest or ones in flight, for which the measurement of the muon momentum [4], combined with information on the pion and muon masses, provides the information. The most stringent existing bound is $m_{\nu_\mu} \leq 0.16$ MeV at 90% C.L.. The polarization of the muon from pion decay is determined from the relative electron rate at the momentum end point in a direction opposite to the $\mu$-spin in $\mu$-decay. This has been used [5] to obtain the neutrino helicity $\mid h_{\nu_\mu} \mid \geq 0.9968$, at a 90% C.L..

Limits on $m_{\nu_\tau}$, the mass of $\nu_\tau$, come from studies of the invariant mass and energy of the hadronic system in $\tau \longrightarrow \nu_\tau +$ hadrons. The bigger the invariant mass is, the less energy is available to make the mass of $\nu_\tau$ in the final state. The most stringent laboratory bound is [6] $m_{\nu_\tau} \leq 24$ MeV at 95% C.L. . Recent analyses [7] show that a tau-neutrino of mass between 1 MeV and 25 MeV can have a host of interesting astrophysical and cosmological consequences. In light of this interest, we consider the $\nu_\tau$-mass effect on the pure-leptonic decay of charmed mesons $D \longrightarrow \tau \ \nu_\tau$ and $D_s \longrightarrow \tau \ \nu_\tau$. The pure-leptonic decays of charm are showing up [8] recently and their evidence and precision will improve over the next few years as large data samples become available. The present knowledge can be summarized [9] in the upper limit on the $D^+$ decay, $B(D^+ \longrightarrow \mu^+\nu_\mu) \leq 7.2 \cdot 10^{-4}$, at a 90% C.L., and the evidence on the $D_s^+$ decay, $B(D_s^+ \longrightarrow \mu^+\nu_\mu) = (5.9 \pm 2.2) \cdot 10^{-3}$. The $\tau^+ \ \nu_\tau$ channels are a factor 2.6 and 9.8 more probable, respectively, but their search is difficult due to the presence of at least two neutrinos in the final state.

For the pure-leptonic decay of the pseudoscalar meson $P^+ \longrightarrow l^+ + \nu_l$, the decay amplitude is given by

$$T_{\lambda,\lambda'} = \langle l^+(k_l, \lambda') \ \nu_l(k_\nu, \lambda) \mid T \mid P^+(q) \rangle = i \ \frac{G_F}{\sqrt{2}} f_P \ q_\mu \ \bar{u}_{\nu_l}(k_\nu, \lambda)\gamma^\mu (1 - \gamma_5) \ v_l(k_l, \lambda')$$

where $f_P$ is the $P$-meson decay coupling constant.

For $m_\nu = 0$ the left-handed chirality of charged current weak interactions then leads to a definite helicity -1 for the outgoing neutrino and, a fortiori, for the $l^+$ in the C.M. frame. However, for $m_\nu \neq 0$ the helicity has no definite value and we talk about the

polarization components of the neutrino. The transition probability for a given helicity $\lambda$ of the neutrino is $\Gamma_\lambda = \mid \vec{k} \mid \mid T_\lambda \mid^2 /(8\pi\, m_P^2)$, where $m_P$ is the meson mass and the squared amplitude, $\mid T_\lambda \mid^2 = \sum_{\lambda'} \mid T_{\lambda\lambda'} \mid^2$, is given by

$$\mid T_\lambda \mid^2 = 2G_F^2 f_P^2 \{2(k_\nu.q)(k_l.q) - (k_\nu.k_l)q^2 - 2\lambda\, m_\nu(k_l.q)(s.q) + \lambda\, m_\nu(k_l.s)q^2\} \quad (1)$$

The momentum $\mid \vec{k} \mid$ in C.M., determined by kinematics, is related to $m_\nu$, $m_\nu^2 = m_P^2 + m_l^2 - 2m_P\sqrt{m_l^2 + \mid \vec{k} \mid^2}$. This method is currently being used for the determination of $m_{\nu_\mu}$ in the $\pi$-decay case. The polarization four-vector $s^\mu$ in Eq (1) satisfies $s.s = -1$ and $s.k_\nu = 0$ and its choice is dictated by the polarization basis. For $m_\nu \neq 0$, in the rest frame of the neutrino, $s^\mu = (0, \vec{s})$ with $\vec{s}$ the unit vector along the chosen direction of quantization.

The neutrino longitudinal polarization can be easily calculated putting $\vec{s}$ in the direction of motion of the neutrino, yielding

$$P_{long} = \frac{(E - W) \mid \vec{k} \mid}{(WE - \mid \vec{k} \mid^2)} \quad (2)$$

where $E$ and $W$ are the energies of the neutrino and charged lepton, respectively.

To obtain the decay rate is staightforward: the sum over the helicity $\lambda$ gives

$$\Gamma = \sum_\lambda \Gamma_\lambda = \frac{G_F^2 f_P^2}{8\pi m_P^2} \mid \vec{k} \mid \left( m_P^2 (m_l + m_\nu)^2 - \left( m_l^2 - m_\nu^2 \right)^2 \right)$$

Having this, we can easily calculate the ratio between the decay rates of a meson to two different leptons, namely, tau lepton and muon, getting an answer independent of $f_P$.

So far we have seen that dynamical effects of a neutrino mass different from zero show up in the leptonic decay of a meson. This effect depends on the mass of the decaying meson as well as on those of the daughter-products. Therefore, situations exist where these effects become more important, and our aim now is to look for them.

We are interested in a longitudinal polarization in C.M. different from -1, it is easy to see that the most favourable situation takes place when little phase space is left. Having this in mind, we must search for a decay where the mass of the charged lepton and the mass of the meson are as close as possible.

For the tau neutrino there is a very favourable situation with the $D$ meson. Another possibility is to use the decay of the $D_s$, in this case more phase space and more $\nu$s are available, although the neutrino mass effects are somewhat diluted. For the muon neutrino the best decay for these purposes is the decay of the pion.

According to what was stated before, the decay $D^+ \longrightarrow \tau^+ + \nu_\tau$ is the best decay to look for dynamical effects of the tau-neutrino mass. This decay has not been observed yet, and we can expect to see it in the future in a tau-charm factory [10]. Its measurement is of interest to determine $f_D$. For our purposes to study the neutrino mass effects, we calculate the ratio between the decays $D^+ \longrightarrow \tau + \nu_\tau$ and $D^+ \longrightarrow \mu + \nu_\mu$ for different masses of the tau-neutrino. The result is given in Table I, showing a 3% effect at most for $m_{\nu_\tau}$ up to 24 MeV. For the longitudinal polarization of the neutrino in the C.M. we can see from Eq. (2) how different from -1 are the results in terms of the neutrino mass. In the best situation (the heaviest possible neutrino compatible with the experimental limit)

the value of $-P_{long}$ differs from one in a 4%. So one should have a good experimental precision for the effect to show up. This is envisageable in a tau-charm factory, measuring the hadron spectrum and angular distribution from tau decay coming from a $D$, even if we cannot reconstruct the tau-direction [11]. Besides that a tau-charm factory has the capability of tightly controlling the background and systematic errors [10].

On the other hand, the $D_s$ decay may be used too as a tool to get experimental evidence of neutrino mass. This decay leaves a bigger phase space, shrinking the numerical predictions as can be seen in Table II: 1% effect at most for both branching ratio and polarization. The statistics is however better.

We turn our attention to the muon-neutrino. The most attractive scenario to look for a hint of $m_{\nu_\mu}$ different from zero is pion decay. This decay has been widely observed and the precision in the muon momenta is strikingly high. We have explored the possibility of using the decay rate and the longitudinal polarization to look for muon-neutrino mass effect. They cannot compete with the kinematical determination.

This analysis shows that the mass effects based on the dynamical observables could be of interest for the $\nu_\tau$. Even more, although the branching ratio is Lorentz invariant, the longitudinal polarization **is not**. The helicity is a Lorentz invariant quantity only for massless particles. In this way, if the neutrino has zero mass, it will have negative helicity in any frame. On the other hand, if we allow the neutrino to have a mass, its speed is no longer 1, so that boosting appropiately the reference frame, i.e. the decaying meson, we can easily generate the opposite helicity [12]. The rest of the paper is going to be devoted to analise neutrino-mass effects in a boosted system.

The effect of a Lorentz transformation on the polarization appears as a rotation of the polarization three-vector relative to the particle three-momentum: this is called a Wigner rotation [3]. Then we can see [13] that the amplitudes for a process $A(q) \to a(k_a, \lambda_a)b(k_b, \lambda_b)$ transform in the following way ,

$$T_{\lambda_a\lambda_b}(q, k_a) = \sum_{\lambda_a'\lambda_b'} (-)^{\lambda_b' - \lambda_b} \, d^{1/2}_{\lambda_a'\lambda_a}(\omega_a) \, d^{1/2}_{\lambda_b'\lambda_b}(\omega_b) \, T_{\lambda_a'\lambda_b'}(q', k_a')$$

where $\omega_a$ and $\omega_b$ are the Wigner rotations for the particles $a$ and $b$ under a boost which transforms their momenta : $k_i' = \Lambda k_i$ . The rotations are as follows,

$$\sin \omega_i = \frac{m_i \sinh \kappa \sin \theta_i'}{|\vec{k_i}|} \quad , \quad \cos \omega_i = \frac{|\vec{k_i'}| \cosh \kappa - E_i' \sinh \kappa \cos \theta_i'}{|\vec{k_i}|} \tag{3}$$

where $\kappa$ is the parameter of the boost related to the velocity by $v = \tanh \kappa$, $E_i'$ the energy of particle $i$ and $\theta'$ the angle between the decaying particle, $A$, and the direction of the final particle in the boosted frame.

We are interested in studying the density matrix of the final neutrino in the boosted frame in a decay $P^+(q) \to \nu_l(k_\nu, \lambda_\nu)l^+(k_l, \lambda_l)$. Then it is easy to see that the density matrix for the neutrino transforms as a direct product of two helicity amplitudes if we average over the polarization of the charged lepton. Our boost will take us from the C.M. frame to the LAB frame, where the meson has three-momentum $|\vec{q'}|$ different from zero. Now we can get the density matrix of the $\nu_l$ in the boosted frame, if we know it in C.M.:

$$\rho^{LAB} = d^{1/2}(\omega) \cdot \rho^{CM} \cdot d^{1/2\,T}(\omega) \tag{4}$$

where $\omega$ is the Wigner rotation asociated with the $\nu_l$.

As can be easily seen, in the C.M. frame we only have longitudinal polarization. We apply the transformation law (4) to get the expression of the density matrix in the LAB frame.

$$\rho^{LAB} = \frac{1}{2} \begin{pmatrix} 1 + P_{long} \cos \omega & P_{long} \sin \omega \\ P_{long} \sin \omega & 1 - P_{long} \cos \omega \end{pmatrix}$$

We can see that, in the new frame of reference, a new transverse polarization has appeared, $P_x = P_{long} \sin \omega$, and the longitudinal polarization has been modified too: $P_z = P_{long} \cos \omega$. This is a very important fact because, as we have already pointed out, if $m_\nu = 0$, in every frame the neutrino will have only longitudinal polarization, and it will always be equal to -1. Moreover, notice that the effects on the transverse polarization are linear in $m_\nu$ as shown in Eq. (3), so transverse polarization will be more sensitive to small neutrino masses.

Let us try to analyse carefully the new situation in the boosted frame. Qualitatively, we have two very different situations:

i) the velocity of the decaying particle in LAB is less than the velocity of $\nu_l$ in C.M.,

ii) the velocity of the decaying particle in LAB is bigger than the velocity of $\nu_l$ in C.M.

In i), in LAB, we still have particles both in the forward and backward hemispheres. Concerning the Wigner rotation we have an important feature: for any value of $\theta'$, $\cos \omega$ will always be positive and consequently $\sin \omega$ will have a maximum value strictly less than 1,

$$\sin \omega|_{max} = \frac{m_\nu \sinh \kappa}{|\vec{k}_\nu|} = \frac{m_\nu \, |\vec{q'}|}{|\vec{k}_\nu| \, m_P}$$

In ii), all the $\nu_l$ are now in the forward hemisphere in LAB. From kinematics, there is a maximum $\theta'$ for which $\sin \omega = 1$. The value of $\cos \omega$ can even change sign for the neutrinos that were in the backward hemisphere in the C.M. frame (which appear forward in the LAB). This means, for instance, that the neutrinos which in C.M. were moving opposite to the boost, have reversed the direction of its momenta and they are now right-handed neutrinos.

As we can see, the effects of a boost on the neutrino polarization are really important. We could ask whether they also translate into an observable effect of the charged lepton polarization in the boosted frame. It would be much easier to measure the charged lepton polarization from the distribution of its decay products than the $\nu_l$ polarization, and we could naively expect them to be somehow related. In fact, in the C.M. frame the charged lepton polarization is exactly opposite to the neutrino polarization, but this is no longer true in a boosted frame, due to the orbital angular momentum in this frame. Because of this, we need to reconstruct completely the decay, this means we have to know the tau and meson directions. Experimentally, this is not an easy task, because one has two neutrinos in each $D$ decay. Therefore, from the practical point of view, unless we have an extremely good experimental precision it is easier to deal with the neutrinos directly. In this letter we are going to follow this avenue. However, one should keep in mind that a complete reconstruction of the decay would provide the information also from the tau.

Taking into account the available and foreseen experimental facilities we choose to take profit of the existing and proposed neutrino beams [14], [15] at FermiLab and LHC. The Achiles' heel of this approach is that we have to measure the neutrino polarization. The transverse polarization is, a priori, the most sensitive one to neutrino mass, but its measurement would be rather complicated and would introduce additional masses. Fortunately, to measure the longitudinal polarization will be much easier.

If we boost our neutrinos according to the situation ii), a measurable fraction of the neutrinos will have reversed their helicity. Neglecting those interactions proportional to neutrino mass, the gauge interactions are left-handed, so that *only* those neutrinos with the *proper* helicity will interact. This means that we have to look for a **left-handed neutrino deficit** in order to reveal these effects.

Specifically, the flux and the corresponding spectrum of boosted neutrinos could be obtained from measurements on the $D$-mesons produced. The dynamical neutrino mass effects discussed here provide different fractions of improper, right-handed, neutrinos as a function of its mass and energy. So a deficit of left-handed neutrinos will be indeed a proof of neutrino mass. Not only this, the amount of such a deficit will be a measure of the neutrino mass itself. The kind of dissapearance will shed light into the character of the neutrino type. Clearly, the track left by a Dirac neutrino would be only a depletion in the expected flux of active neutrinos due to the sterile nature of right-handed neutrinos. However, a Majorana neutrino would bring in its right-handed state new reactions associated with the charge conjugated state. Summing up, counting neutrino interations provides a measure of its longitudinal polarization.

Following this strategy, we have to calculate the fraction of left-handed neutrinos in the beam. To do so, we need the neutrino spectrum which is given by $d\Gamma/dE_\nu = (16\pi E_{q'} \mid \vec{q'} \mid)^{-1}$, at a fixed boost velocity, where $E_{q'}$ is the parent meson energy in LAB. As we need the spectrum of left-handed neutrinos, we just have to multiply the energy spectrum by the fraction of left-handed neutrinos $f(L)$ at a given energy

$$\frac{d\Gamma_L}{dE_\nu} = \frac{d\Gamma}{dE_\nu} f(L) \quad ; \quad f(L) = \frac{1 - P_z(E_\nu)}{2}$$

where $P_z(E_\nu) = P_{long} \cos \omega$ is the neutrino longitudinal polarization in the boosted frame at a given energy.

Figure 1 shows the fraction of left-handed neutrinos as a function of the neutrino energy for a fixed boost (100 GeV) of the $D$ (a) and $D_s$ (b) mesons and a neutrino mass of 24 MeV. As we can see, the effect is huge for the low energy neutrinos and it dilutes for increasing energy.

In Figure 2 we present the integrated spectrum of left neutrinos as a function of the boost energy, normalized to the total flux of neutrinos, i.e, one minus the deficit. It can be seen that above a boost of moderate energy around 20 GeV, the deficit becomes constant and we do not improve the integrated effect by increasing the boost energy. However, the energy of the improper helicity neutrinos increases with that of the boost. This is very important if a cut in threshold energy is applied as discussed now.

According to Figure 2, for a boost above 20 GeV we can expect a depletion of a 7.5% for the $D$ decay and a tau-neutrino of 24 MeV, if we integrate the complete spectrum. This roughly quadruplicates what we can get in C.M. measuring the tau polarization.

When considering the threshold (2 GeV) in charged current reactions for detecting tau-neutrinos we get a sizable deficit of left-neutrinos for high velocity boosts: at 100 GeV we find an integrated depletion of 4% above 2 GeV, and it increases with the boost towards the asymptotic 7.5%. This is so because the fraction of wrong helicity neutrinos, which is bigger in the low energy part of the spectrum, moves upwards with the boost. The deficit fraction would increase for neutrinos below some energy cut, say 1 or 2 GeV, reaching 36% or 23%, respectively for a boost of 100 GeV. For tau neutrinos, these low energy neutrinos are only detectable through neutral current scattering. Even through neutral current interactions the distinction between Dirac or Majorana can be made [16]. Neutral current events are dominated by $\nu_\mu$ events, however these can be controlled and substracted by measuring the charged current events at the same energy they give rise to. The best strategy for the $\nu_\tau$-detection will depend on the actual characteristics of neutrino beams [14], [15]. For boosts high enough, charged current detection is simpler and the number of counts increases linearly with the energy. In particular, for Majorana neutrinos, the presence of a wrong sign charged lepton constitutes a clean signature.

The situation is not so promising for the $D_s$ decay. In this case, we get a deficit of approximately 2.5% when we integrate the complete spectrum. Putting an upper cut in the energy, in this case, gives a deficit of 22% (1 GeV) or 13% (2 GeV), again for a boost of 100 GeV.

To conclude: in this work we have studied the viability of a dynamical determination of the tau neutrino mass using branching ratios and longitudinal polarizations, instead of the direct kinematical approach. In light of kinematics, the most favourable situation is the $D$-meson decay which allows us to get effects of at most 4% in the C.M. longitudinal polarization. The situation becomes more promising if we take advantage of the fact that, while the branching ratio is a Lorentz-invariant quantity, the longitudinal polarization is not. Under appropriate conditions, the polarization can be dramatically modified by a boost. We have discussed possible strategies to observe the enhanced deficit of left-neutrinos.

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## Table I

| $m_{\nu_\tau}$ ( MeV) | 0 | 6 | 12 | 18 | 24 |
|---|---|---|---|---|---|
| $-P_{long}$ | 1 | .9975 | .9901 | .9778 | .9603 |
| $\frac{\Gamma(D\to\tau\,\nu_\tau)}{\Gamma(D\to\mu\,\nu_\mu)}$ | 2.640 | 2.636 | 2.622 | 2.598 | 2.563 |

## Table II

| $m_{\nu_\tau}$ ( MeV) | 0 | 6 | 12 | 18 | 24 |
|---|---|---|---|---|---|
| $-P_{long}$ | 1 | .9993 | .9973 | .9940 | .9893 |
| $\frac{\Gamma(D_s\to\tau\,\nu_\tau)}{\Gamma(D_s\to\mu\,\nu_\mu)}$ | 9.739 | 9.736 | 9.726 | 9.710 | 9.687 |

## Table captions

**Table I :** CM frame observables for the decay $D \to \tau\,\nu_\tau$. The first row shows the neutrino-tau mass effects in the longitudinal polarization, while the second row shows the branching ratio between the decay rates of the $D$ meson to tau and muon for different masses of the tau-neutrino (the muon-neutrino is taken to be massless).

**Table II :** As Table I, but for the decay $D_s \to \tau\,\nu_\tau$.

## Figure captions

**Figure 1 :** Fraction of left-handed neutrinos as a function of their energy for a neutrino mass of 24 MeV and a boost of 100 GeV in the $D$ (a) and $D_s$ (b) decays.

**Figure 2 :** Left-handed fraction of the total neutrino flux, $R = \Gamma_L/\Gamma$, as a function of the parent meson boost for $D$ (solid line) and $D_s$ (dashed line) decays.
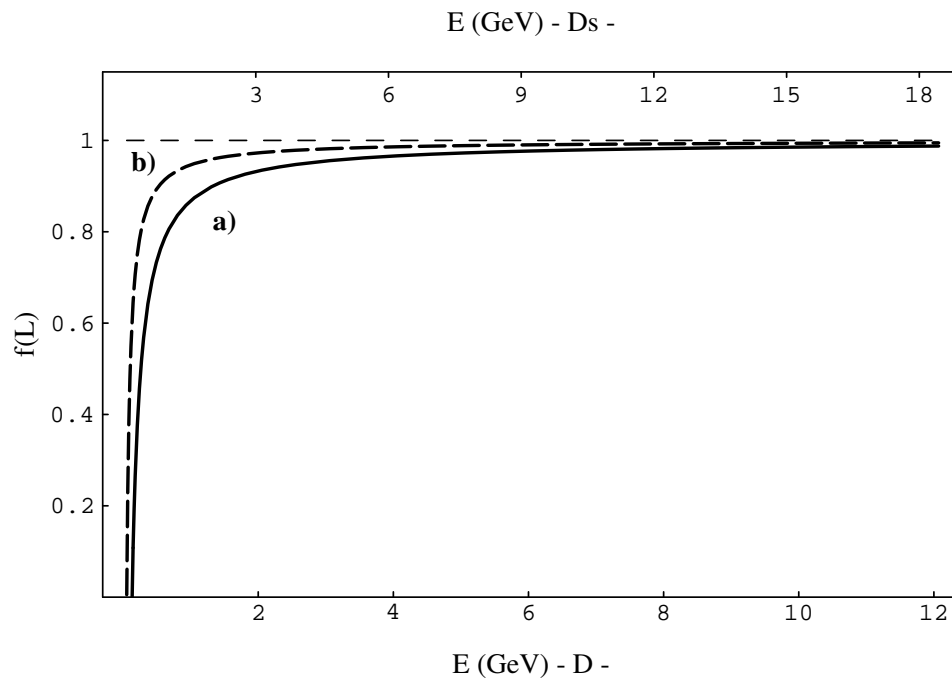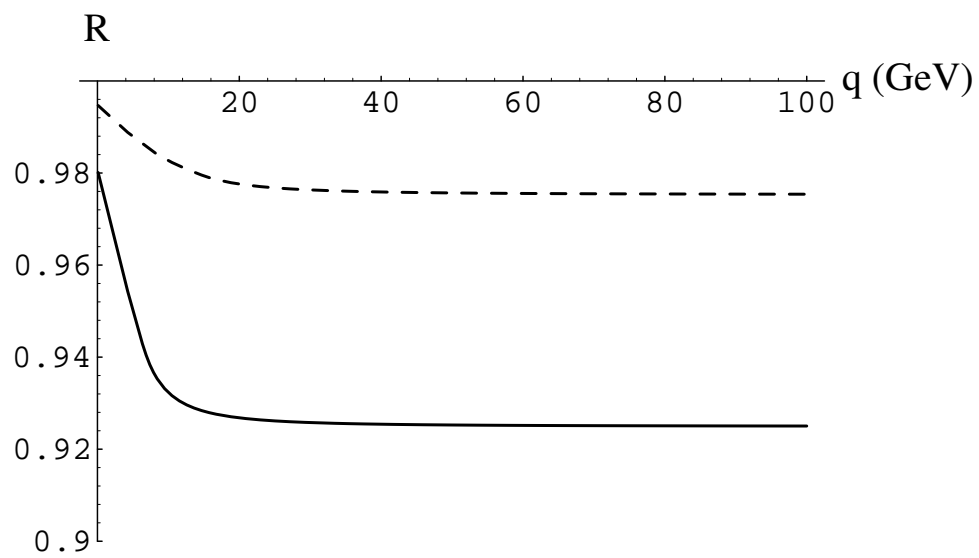
E (GeV) - Ds -

f(L)

E (GeV) - D -

Figure 1

Figure 2